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Finite temperature effects on axions in the early universe

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Abstract
The impact of finite temperature effects on axions in the context of cosmology is studied here for two interesting cases. We find that they may be significant for detailed and precise abundance and rate calculations of axions in the early universe.

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1. Introduction

Axions, the pseudo Goldstone bosons, associated with the Peccei–Quinn (PQ) symmetry, $U(1)_{\mathrm{PQ}}$, remain the most popular solution to the strong CP problem [1–4]. Axions, QCD type or some other variant, appear in many extensions of the standard model. The mass and coupling of the axion is inversely proportional to the scale at which $U(1)_{\mathrm{PQ}}$ is broken, denoted by f_a, and hereafter referred to as the PQ scale or axion decay constant interchangeably. This has important implications for axion cosmology because axions can only be produced thermally when the temperature of the universe drops below f_a. The PQ scale, f_a, is constrained by astrophysics, cosmology and laboratory experiments. For a review of latest bounds on f_a, refer to [5, 6]. More recently, results from the PVLAS experiment [7] indicate the existence of an axion-like particle, though at the moment it is difficult to reconcile any standard axion-like particle with these results. Some of the possible ways out of this difficulty are discussed in [8].

In this paper, we would like to investigate some cosmological implications/effects in the context of axions by taking into account the finite temperature effects on axion decay constant while discussing issues such as relic axion abundance etc. We intend to explore the possible impact of such corrections on axion physics at different epochs of the early universe. Mostly sharing the basic properties with the pions, notably, the axion coupling to two gluons is given by the anomaly term (the two photon coupling has an analogous form)

$$L = g_{agg}\epsilon_{\mu\nu\rho\sigma}G^{b\mu\nu}G^{b\rho\sigma}a, \tag{1}$$

where $\tilde{G}^b_{\mu\nu}$ is the dual of the gluon field defined as

$$\tilde{G}^b_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{b\rho\sigma}, \tag{2}$$

where b is the colour index and g_{agg} is the axion coupling which is inversely proportional to f_a. The axions can be further divided into two categories: (a) hadronic axions which couple to quarks only and (b) non-hadronic axions having a coupling to the leptons.

Having noted that the axion coupling to massless gauge bosons is the same as pion–photon–photon coupling, we model the temperature effects in an analogous way as well. To this end, we first note that the results on chiral dynamics and anomaly at finite temperature [9] imply the following:

$$f_\pi(T) = \left(1 - \frac{1}{12}\frac{T^2}{f_\pi^2}\right) f_\pi, \tag{3}$$

where $f_\pi \equiv f_\pi(T=0) \sim 93$ MeV is the zero temperature pion decay constant measured in the laboratory. It is important to keep in mind that such a result is valid only for $T < f_\pi$. In the case of axions, it is natural to look at temperatures lower than the axion decay constant as it is only then that the axions can be thermally produced. Therefore, the above result for the pions can be directly extended to the axion case. The analysis in [9] however brings out a crucial point. The effective pion–photon–photon coupling is inversely proportional to the decay constant and one would have expected that the coupling changes the way dictated by the change in decay constant with the temperature, i.e. one would have expected the coupling to change as $g_{\pi\gamma\gamma} \to g_{\pi\gamma\gamma}\left(1 - \frac{1}{12}\frac{T^2}{f_\pi^2}\right)^{-1}$ or that to $\mathcal{O}\left(T^2/f_\pi^2\right)$, $g_{\pi\gamma\gamma}(T) \sim 1/f_\pi(T)$. What is found instead is that to this order $g_{\pi\gamma\gamma}(T) \sim f_\pi(T)$, i.e.

$$g_{\pi\gamma\gamma}(T) = \left(1 - \frac{1}{12}\frac{T^2}{f_\pi^2}\right) g_{\pi\gamma\gamma}. \tag{4}$$

We therefore take this form for the axion–gluon–gluon coupling and work with this. One may wonder whether similar results can be expected for axion coupling to photons. The reason for this worry is the fact that the pion triplet has an underlying chiral dynamics while there is no such thing for an axion. However, what is relevant here for our purpose is just the calculation of the triangle diagram at finite temperature and barring the model dependent, order unity, effects, the results for the AVV triangle diagram should remain the same.

Further, we have for the pion mass (again in the regime $T < f_\pi$),

$$m_\pi(T) = m_\pi\left(1 + \frac{T^2}{24 f_\pi^2}\right). \tag{5}$$

First, we study axion thermalization and abundance in the very early universe including the temperature effects. The second case investigated is the axion hadron interactions in the post-QCD era.

2. Axion thermalization and abundance

As mentioned earlier, axions can be thermally produced in the universe once the temperature falls below the axion decay constant. In [10], the authors identify the conditions under which there is significant production and/or thermalization of axions. We therefore closely follow

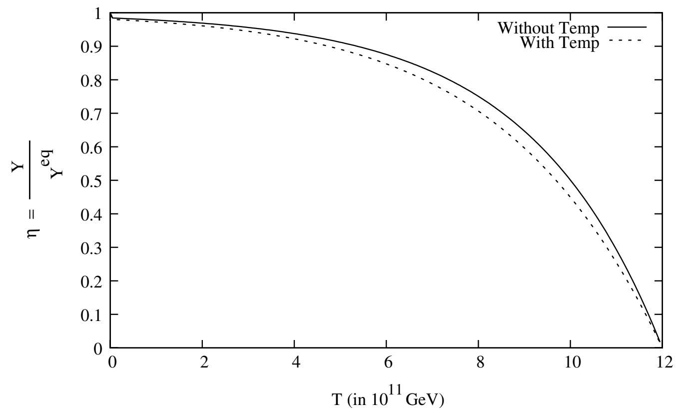

Figure 1. Abundance of axions for $f_a = 1.2 \times 10^{12}$ GeV.

[10], but now include temperature effects as mentioned before. The main thermalization processes are axion–gluon and axion–quark scatterings. The Boltzmann equation for the abundance of axions in an expanding universe is given by

$$x\frac{\mathrm{d}Y}{\mathrm{d}x} = \frac{\Gamma}{H}(Y^{\mathrm{eq}} - Y), \tag{6}$$

where $x = \frac{f_a}{T}$, H is the Hubble expansion rate and Γ is the thermally averaged rate of reaction of axion processes. In the radiation-dominated era, the Friedmann equation yields

$$H = \left(\frac{4\pi^3 g_{\mathrm{eff}}}{45}\right)^{1/2} \frac{T^2}{M_P}, \tag{7}$$

where g_{eff} is the effective degrees of freedom at the temperature T. Compared to the expression for the thermal averaged interaction rate in [10], the rate now looks after including temperature-dependent axion–gluon coupling

$$\Gamma \simeq 7.1 \times 10^{-6}\frac{T^3}{f_a^2}\left(1 - \frac{T^2}{12 f_a^2}\right)^2 = \Gamma_0\left(1 - \frac{T^2}{12 f_a^2}\right)^2. \tag{8}$$

Y^{eq} is the abundance when axions are in thermal equilibrium with other SM particles. In terms of variables $\eta = \frac{Y}{Y^{\mathrm{eq}}}$ and $k = x\frac{\Gamma_0}{H}$, equation (6) becomes

$$x^2\frac{\mathrm{d}\eta}{\mathrm{d}x} = k\left(1 - \frac{1}{12x^2}\right)^2 (1 - \eta). \tag{9}$$

Then we plot the solution to equation (9), as shown in figure 1, which compares the axion abundance with and without the temperature effects taken into consideration for $f_a = 1.2 \times 10^{12}$ GeV. We see that there is a perceptible change in the abundance for the temperature range $(6\text{–}10)\times 10^{11}$ GeV.

3. The hadronic axion in the post-QCD era

As the name suggests the hadronic axion has no lowest level coupling to the charged leptons, and therefore the induced coupling is expected to be small enough to be neglected altogether. For updated bounds on hadronic axion in this context, refer to [11]. Although the exact limits

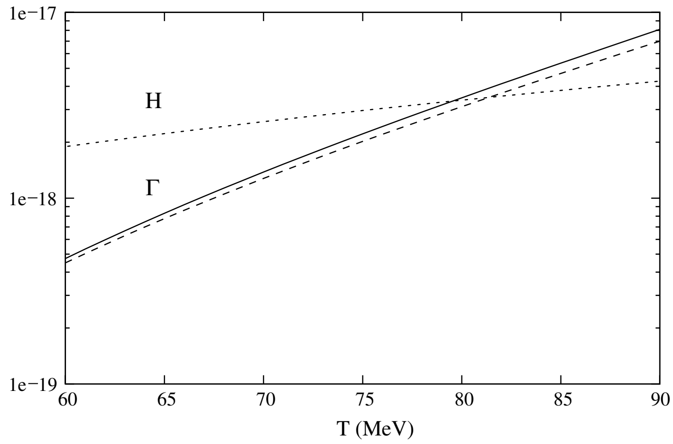

Figure 2. Hadronic axion reaction rate with (solid) and without (dashed) temperature effect for $f_a = 10^7$ GeV.

on the mass and decay constant of the axion are model dependent, up to order unity factors, the limits from various sources suggest $f_a \geqslant 0.6 \times 10^9$ GeV and $m_a \leqslant 0.01$ eV. The axion coupling to nucleons is constrained by the observed neutrino signal from Supernovae 1987A. Supernovae observations give stringent constraints on the mass and coupling: $m_a \geqslant 0.01$ eV or $f_a \leqslant 0.6 \times 10^9$ GeV. However it still leaves a domain for hadronic axion, where f_a lies in the range 3×10^5 GeV to 3×10^6 GeV [12]. But this range is disfavoured by a detailed combined analysis of hadronic axion with cosmological data such as large scale structure, cosmic microwave background, supernova luminosity distances, Lyman-α forest and Hubble parameter [11], which obtains the following limits: $m_a < 1.05$ eV or equivalently $f_a > 5.7 \times 10^6$ GeV. However, in the post-QCD era of interest to us here, pion–axion interaction is important for thermalization and decoupling of these hadronic axions in this era is verified in [12].

The axion–pion interaction is of the form [12]

$$L_{a\pi} = \frac{C_{a\pi}}{f_\pi f_a}(\pi^0\pi^+\partial_\mu\pi^- + \pi^0\pi^-\partial_\mu\pi^+ - 2\pi^+\pi^-\partial_\mu\pi^0)\partial_\mu a. \tag{10}$$

In hadronic axion models, the coupling constant is

$$C_{a\pi} = \frac{1-z}{3(1+z)}. \tag{11}$$

The relevant processes are $a\pi^\pm \to \pi^0\pi^\pm$ and $a\pi^0 \to \pi^+\pi^-$. Following [11, 12], we calculate the axion decoupling temperature in this context but now taking the finite temperature effects in the pion decay constant and mass into account given in equations (3) and (5) respectively (valid for $T < f_\pi$). The decoupling temperature is that where the Hubble expansion rate equals the thermally averaged interaction rate. In figure 2, we plot the expansion rate of the universe (H) and the interaction rates (Γ)—with and without temperature effects put in. The points where the Γ curves cut the H curve give the decoupling temperature. This is shown in the figure for $f_a = 10^7$ GeV. It is clear from the figure that the decoupling temperature is lowered once the finite temperature effects are included. From the figure, it can be seen that the difference in the two cases is not very large. We also compare decoupling temperature of axions in the present calculation with the previous ones (without

Table 1. Decoupling temperature of axions T_{D1} (without) and T_{D2} (with) temperature effects for different values of f_a.

f_a (GeV)	T_{D1} (MeV)	T_{D2} (MeV)
3×10^5	26.43	26.43
1×10^6	35.34	35.34
3×10^6	49.84	49.5
1×10^7	81.04	79.12
1.2×10^7	87.61	85.48
1.3×10^7	90.1	87.9

the temperature effects) for several representative values of f_a in table 1. We find that for higher values of f_a, the temperature effects start becoming significant. However, within the approximation adopted here, it is not possible to explore the range $f_a \geqslant 1.5 \times 10^7$ GeV as the temperatures exceed the pion decay constant, thus taking us away from the validity of the approximation employed. It is therefore important to investigate the temperature effects going beyond this approximation and exploring the consequences.

4. Discussion

We have investigated the impact of temperature effects, though in a limited sense, on the axion cosmology. We have focused our attention of the temperature effects on the axion decay constant, pion decay constant and pion mass in two important epochs of the universe. The results show that there can be perceptible differences in the predictions of abundances and decoupling temperatures. However, we must remark that these effects have been considered in the approximation when the temperature of the universe is lower than the decay constant(s). In the case of axion thermalization in the early universe, we see some change in the abundance of the axions with temperature. In the case of axion interacting with pions in the post-QCD era, we find that temperature effects in the pion mass and decay constant lead to a lower decoupling temperature for the axions. We find that as the axion decay constant is increased, the difference between the zero temperature and finite temperature cases starts becoming significant. However, we have worked in the limit when temperatures are smaller than f_π. This puts a natural limit to the region that we can explore. It is worthwhile to try to explore what happens when $T > f_\pi$. This will require going beyond the validity of the present calculation. We would also like to mention that a more detailed and consistent calculation requires computing the matrix elements including thermal loops also to the same order in temperature corrections and then studying the full impact. In particular, it is important to investigate in detail whether the presence of heat bath (which breaks Lorentz symmetry) can generate any large new couplings or whether all the effects are already captured in modification of couplings and masses at finite temperature.

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